Write your name here

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**Pearson Edexcel GCE**

Centre Number

Candidate Number

# AS and A level Further Mathematics
# Core Pure Mathematics

# Practice Paper
# Matrix algebra (part 1)

**You must have:**
Mathematical Formulae and Statistical Tables (Pink)

Total Marks

## Instructions

- Use black ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- Fill in the boxes at the top of this page with your name, centre number and candidate number.
- Answer all the questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – there may be more space than you need.
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

## Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 13 questions in this question paper. The total mark for this paper is 100.
- The marks for each question are shown in brackets – use this as a guide as to how much time to spend on each question.
- Calculators must not be used for questions marked with a * sign.

## Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

**1.**

$$\mathbf{M} = \begin{pmatrix} x & x-2 \\ 3x-6 & 4x-11 \end{pmatrix}$$

Given that the matrix **M** is singular, find the possible values of $x$.

**(Total 4 marks)**

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**2.**

$$\mathbf{A} = \begin{pmatrix} 2 & -1 \\ 4 & 3 \end{pmatrix}, \qquad \mathbf{P} = \begin{pmatrix} 3 & 6 \\ 11 & -8 \end{pmatrix}$$

(*a*) Find $\mathbf{A}^{-1}$

**(2)**

The transformation represented by the matrix **B** followed by the transformation represented by the matrix **A** is equivalent to the transformation represented by the matrix **P**.

(*b*) Find **B**, giving your answer in its simplest form.

**(3)**

**(Total 5 marks)**

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**3.** (i) $$\mathbf{A} = \begin{pmatrix} 2k+1 & k \\ -3 & -5 \end{pmatrix}, \text{ where } k \text{ is a constant}$$

Given that $$\mathbf{B} = \mathbf{A} + 3\mathbf{I}$$

where **I** is the $2 \times 2$ identity matrix, find

(*a*) **B** in terms of $k$,

**(2)**

(*b*) the value of $k$ for which **B** is singular.

**(2)**

(ii) Given that $$\mathbf{C} = \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix}, \quad \mathbf{D} = \begin{pmatrix} 2 & -1 & 5 \end{pmatrix}$$

and $$\mathbf{E} = \mathbf{CD}$$

find **E**.

**(2)**

**(Total 6 marks)**

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**4.** (*a*) Given that $$\mathbf{A} = \begin{pmatrix} 3 & 1 & 3 \\ 4 & 5 & 5 \end{pmatrix} \text{ and } \mathbf{B} = \begin{pmatrix} 1 & 1 \\ 1 & 2 \\ 0 & -1 \end{pmatrix},$$

find **AB**.

**(2)**

(*b*) Given that $$\mathbf{C} = \begin{pmatrix} 3 & 2 \\ 8 & 6 \end{pmatrix} \text{ and } \mathbf{D} = \begin{pmatrix} 5 & 2k \\ 4 & k \end{pmatrix}, \text{ where } k \text{ is a constant}$$

and $$\mathbf{E} = \mathbf{C} + \mathbf{D},$$

find the value of $k$ for which **E** has no inverse.

**(4)**

**(Total 6 marks)**

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**Turn over**

**5.** $$\mathbf{A} = \begin{pmatrix} 2 & 0 \\ 5 & 3 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} -3 & -1 \\ 5 & 2 \end{pmatrix}$$

(*a*) Find **AB**.

**(3)**

Given that $$\mathbf{C} = \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}$$

(*b*) describe fully the geometrical transformation represented by **C**,

**(2)**

(*c*) write down $\mathbf{C}^{100}$.

**(1)**

**(Total 6 marks)**

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**6.** (*a*) Given that $$\mathbf{A} = \begin{pmatrix} 1 & \sqrt{2} \\ \sqrt{2} & -1 \end{pmatrix},$$

(i) find $\mathbf{A}^2$,

(ii) describe fully the geometrical transformation represented by $\mathbf{A}^2$.

**(4)**

(*b*) Given that $$\mathbf{B} = \begin{pmatrix} 0 & -1 \\ -1 & 0 \end{pmatrix},$$

describe fully the geometrical transformation represented by **B**.

**(2)**

(*c*) Given that $$\mathbf{C} = \begin{pmatrix} k+1 & 12 \\ k & 9 \end{pmatrix},$$

where $k$ is a constant, find the value of $k$ for which the matrix **C** is singular.

**(3)**

**(Total 9 marks)**

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**7.** (i) Given that $\mathbf{A} = \begin{pmatrix} 1 & 2 \\ 3 & -1 \\ 4 & 5 \end{pmatrix}$ and $\mathbf{B} = \begin{pmatrix} 2 & -1 & 4 \\ 1 & 3 & 1 \end{pmatrix}$,

(*a*) find **AB**.

(*b*) Explain why $\mathbf{AB} \neq \mathbf{BA}$.

**(4)**

(ii) Given that $\mathbf{C} = \begin{pmatrix} 2k & -2 \\ 3 & k \end{pmatrix}$, where $k$ is a real number

find $\mathbf{C}^{-1}$, giving your answer in terms of $k$.

**(3)**

**(Total 7 marks)**

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**8.** The transformation $U$, represented by the $2 \times 2$ matrix **P**, is a rotation through 90° anticlockwise about the origin.

(*a*) Write down the matrix **P**.

**(1)**

The transformation $V$, represented by the $2 \times 2$ matrix **Q**, is a reflection in the line $y = -x$.

(*b*) Write down the matrix **Q**.

**(1)**

Given that $U$ followed by $V$ is transformation $T$, which is represented by the matrix **R**,

(*c*) express **R** in terms of **P** and **Q**,

**(1)**

(*d*) find the matrix **R**,

**(2)**

(*e*) give a full geometrical description of $T$ as a single transformation.

**(2)**

**Total 7 marks)**

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**9.** A right angled triangle $T$ has vertices $A(1, 1)$, $B(2, 1)$ and $C(2, 4)$. When $T$ is transformed by the matrix $\mathbf{P} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$, the image is $T'$.

(*a*) Find the coordinates of the vertices of $T'$.

**(2)**

(*b*) Describe fully the transformation represented by **P**.

**(2)**

The matrices $\mathbf{Q} = \begin{pmatrix} 4 & -2 \\ 3 & -1 \end{pmatrix}$ and $\mathbf{R} = \begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$ represent two transformations. When $T$ is transformed by the matrix **QR**, the image is $T''$.

(*c*) Find **QR**.

**(2)**

(*d*) Find the determinant of **QR**.

**(2)**

(*e*) Using your answer to part (*d*), find the area of $T''$.

**(3)**

**(Total 11 marks)**

**10.** (i)

$$\mathbf{A} = \begin{pmatrix} p & 2 \\ 3 & p \end{pmatrix}, \qquad \mathbf{B} = \begin{pmatrix} -5 & 4 \\ 6 & -5 \end{pmatrix}$$

where $p$ is a constant.

(*a*) Find, in terms of $p$, the matrix $\mathbf{AB}$

**(2)**

Given that

$$\mathbf{AB} + 2\mathbf{A} = k\mathbf{I}$$

where $k$ is a constant and $\mathbf{I}$ is the $2 \times 2$ identity matrix,

(*b*) find the value of $p$ and the value of $k$.

**(4)**

(ii)

$$\mathbf{M} = \begin{pmatrix} a & -9 \\ 1 & 2 \end{pmatrix}, \text{ where } a \text{ is a real constant}$$

Triangle $T$ has an area of 15 square units.

Triangle $T$ is transformed to the triangle $T'$ by the transformation represented by the matrix $\mathbf{M}$.

Given that the area of triangle $T'$ is 270 square units, find the possible values of $a$.

**(5)**

**(Total 11 marks)**

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**11.** $$\mathbf{A} = \begin{pmatrix} -4 & a \\ b & -2 \end{pmatrix}, \text{ where } a \text{ and } b \text{ are constants.}$$

Given that the matrix $\mathbf{A}$ maps the point with coordinates (4, 6) onto the point with coordinates (2, −8),

(*a*) find the value of $a$ and the value of $b$.

**(4)**

A quadrilateral $R$ has area 30 square units.

It is transformed into another quadrilateral $S$ by the matrix $\mathbf{A}$.

Using your values of $a$ and $b$,

(*b*) find the area of quadrilateral $S$.

**(4)**

**(Total 8 marks)**

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**12.**

$$\mathbf{P} = \begin{pmatrix} -\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \\ \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} \end{pmatrix}.$$

(*a*) Describe fully the single geometrical transformation $U$ represented by the matrix **P**.

**(2)**

The transformation $U$ maps the point $A$, with coordinates $(p, q)$, onto the point $B$, with coordinates $(6\sqrt{2}, 3\sqrt{2})$.

(*b*) Find the value of $p$ and the value of $q$.

**(3)**

The transformation $V$, represented by the $2 \times 2$ matrix **Q**, is a reflection in the line with equation $y = x$.

(*c*) Write down the matrix **Q**.

**(1)**

The transformation $U$ followed by the transformation $V$ is the transformation $T$. The transformation $T$ is represented by the matrix **R**.

(*d*) Find the matrix **R**.

**(3)**

(*e*) Deduce that the transformation $T$ is self-inverse.

**(1)**

**(Total 10 marks)**

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**13.**

$$\mathbf{A} = \begin{pmatrix} 2 & 1 \\ -1 & 0 \end{pmatrix} \text{ and } \mathbf{B} = \begin{pmatrix} -1 & 1 \\ 0 & 1 \end{pmatrix}$$

Given that $\mathbf{M} = (\mathbf{A} + \mathbf{B})(2\mathbf{A} - \mathbf{B})$,

(*a*) calculate the matrix **M**,

**(6)**

(*b*) find the matrix **C** such that $\mathbf{MC} = \mathbf{A}$.

**(4)**

**(Total 10 marks)**

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**TOTAL FOR PAPER: 100 MARKS**